Chapter 1 **ALGEBRA OF MATRICES**

1.1 MATRIX

DEFINITION A set of mn numbers arranged in the form of a rectangular *array of m rows and n columns* is called an $m \times n$ matrix (to be read as 'm' by 'n' matrix).

An $m \times n$ matrix is usually written as

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{1j} & a_{1n} \\ a_{21} & a_{22} & a_{23} & ...a_{2j} & a_{2n} \\ a_{ai1} & a_{i2} & a_{i3} & a_{ij} & a_{in} \\ : & ; & : & : & : \\ a_{m1} & a_{m2} & a_{m3} & a_{mj} & a_{mn} \end{bmatrix}_{m \times n}$$

In compact form the above matrix is represented by

$$A = \lfloor a_{ij} \rfloor_{m \times n} \quad \text{or} \quad A = \lfloor a_{ij} \rfloor$$

The numbers a_{11}, a_{12},...etc. are known as **Elements** of the matrix **A.** The element a_{ij} belongs to i^{th} row and j^{th} column and is called the $(i, j)^{th}$ element of the matrix A=$[a_{ij}]$. Thus, in the element a_{ij} the first subscript I always denotes the number of rows and the second subscript j, number of columns in which the element occurs.

Following are some examples of matrices

(i) $A = \begin{bmatrix} 2 & 3 & 8 \\ 1 & -6 & 3 \\ 4 & 0 & 3 \end{bmatrix}$ is a matrix having 3 rows and 3 columns. So the order of the matrix is = 3×3 such that a_{11}=2, a_{12}=3, a_{13}=8, a_{21}=1, $a_{22} = -6$, $a_{23} = 3$, $a_{31} = 4$, $a_{31} = 0$, $a_{33} = 3$.

(ii) $B=\begin{bmatrix}\sin x & \cos x\\ \cos x & -\sin x\end{bmatrix}$ is a matrix having 2 rows and 2 columns. So the order of the matrix is = 3×3 such that $a_{11}=\sin x$, $a_{12}=\cos x$, $a_{21}=\cos x$, $a_{22}=-\sin x$

Types of Matrix

1. **Row Matrix**
   A matrix having only one row is called a ***row matrix.***

   e.g. $A=\begin{bmatrix}1 & -3 & 0 & 4\end{bmatrix}_{1\times 4}$ is a row matrix of order 1×4

2. **Column Matrix**
   A matrix having only one column is called a ***column matrix.***

   e.g. $B=\begin{bmatrix}2\\ -1\\ 3\end{bmatrix}_{3\times 1}$ is a row matrix of order 3×1

3. **Square Matrix**
   In these square matrix, the number of rows is equal to the number of columns.

   e.g. $\begin{bmatrix}1 & 2\\ 5 & 8\end{bmatrix}_{2\times 2}$ and $\begin{bmatrix}1 & 1 & 3\\ 4 & 0 & 3\\ 5 & -1 & 8\end{bmatrix}_{3\times 3}$

In these square matrices, m=n

Elements a_{ij} of a square matrix for which i=j are called ***diagonal elements*** and line along which they lie is called ***principal diagonal*** of the matrix.

4. **Null Matrix or Zero Matrix**

The $m \times n$ matrix whose elements are all 0 is called ***null matrix (**or **zero matrix)*** of the type $m \times n$. It is denoted by **O.**

e.g. $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}_{3X4}$ is zero matrix of type 3×4.

5. **Diagonal Matrix**

A square is called ***diagonal matrix*** if elements above and below the principal diagonal are all zero.

e.g. $A = \begin{bmatrix} a_{11} & 0 & 0 & 0 \\ 0 & a_{22} & 0 & 0 \\ 0 & 0 & a_{33} & 0 \\ 0 & 0 & 0 & a_{44} \end{bmatrix}$

is a diagonal matrix , to be denoted by A= diag [$a_{11}, a_{22}, a_{33}, a_{44}$].

6. **Scalar Matrix**

A diagonal matrix in which all the diagonal elements are equal is called ***scalar matrix.***

e.g. $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}_{3\times3}$ is a scalar matrix of order 3.

7. Identity Matrix or Unit Matrix

A square matrix each of whose diagonal elements is 1 and each of whose non-diagonal elements is equal to 0 is called a ***identity matrix*** or ***unit matrix.*** It is denoted by $\mathbf{I}_n$ when n is order of the matrix.

e.g. $\mathbf{I}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times3}$, $I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4\times4}$ are unit matrices of order

2,3, 4 respectively.

8. Triangular Matrix

It is of two types

(a). Upper Triangular Matrix

In this square matrix, all the elements below the principal diagonal are zero.

e.g. $\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ 0 & a_{22} & a_{23} & a_{24} \\ 0 & 0 & a_{33} & a_{34} \\ 0 & 0 & 0 & a_{44} \end{bmatrix}$

(b) Lower Triangular Matrix

In this square matrix, all the elements above the principal diagonal are zero.

e.g. $\begin{bmatrix} a_{11} & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 \\ a_{31} & a_{32} & a_{33} & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$